Obtaining modular units via a recurrence relation.

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The modular curve Y1(N) parametrises pairs (E,P), where E is an elliptic curve and P is a point of order N on E, and we call an algebraic function on this affine curve a unit if it has no poles or zeroes.

We first review how the recurrence relation of elliptic divisibility sequences gives rise to defining equations for the curves Y1(N). We then show that the same recurrence relation also gives explicit algebraic formulae for a basis of the group of units on Y1(N), and we generalise to Y(N).

This result proves and generalises a conjecture of Maarten Derickx and Mark van Hoeij.