Shadow lines in the arithmetic of elliptic curves

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Abstract: Let E/Q be an elliptic curve and let p be a prime of good ordinary reduction. For every imaginary quadratic field K satisfying the Heegner hypothesis for E, one defines the subspace of p-anticyclotomic universal norms inside E(K)\otimes Q_p. Given certain assumptions on E and p, the p-anticyclotomic universal norms form a one-dimensional vector space, known as the "shadow line" attached to the triple (E,K,p). The Sign Conjecture predicts that the shadow line lies in the larger of the two eigenspaces for complex conjugation on E(K)\otimes Q_p. The Sign Conjecture would follow if one could prove that the p-anticyclotomic height pairing is "as non-degenerate as possible", as predicted by Mazur.

In the case where E/Q has analytic rank 2, E/K has analytic rank 3, and p splits in K/Q, we give an algorithm which computes the p-anticyclotomic height pairing. We use it to give an algorithm which computes shadow lines. In our case, the Sign Conjecture predicts that shadow lines lie in E(Q)\otimes Q_p. Our algorithms may be used to gather evidence for the Sign Conjecture, as well as to investigate a question of Mazur and Rubin regarding the distribution of shadow lines as K varies.

This is joint work with J. Balakrishnan, M. Çiperiani, J. Lang and B. Mirza.