

Torsion subgroups of rational elliptic curves over the compositum of all cubic fields

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Let E be an elliptic curve over \mathbb{Q} and let K be the compositum of all cubic extensions of \mathbb{Q} . I will describe joint work with Harris Daniels, Alvaro Lozano-Robledo, and Filip Najman in which we show that the torsion subgroup of $E(K)$ is finite and determine 20 possibilities for its structure, along with a complete description of the $\overline{\mathbb{Q}}$ -isomorphism classes of elliptic curves that fall into each case. We provide explicit parametrizations for each of the 16 torsion structures that occur for infinitely many $\overline{\mathbb{Q}}$ -isomorphism classes of elliptic curves over \mathbb{Q} , and a complete list of j -invariants for each of the 4 that do not.

This classification involved the explicit computation of various modular curves of level dividing 108 and genus up to 4, as well as the determination of their rational points. I will highlight some of the techniques that we used to do this.