

Analytic Questions Concerning Shintani Zeta Functions

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The Shintani zeta function is the Dirichlet series counting cubic rings, and its analytic properties can be used to study the distribution of cubic fields. It is also interesting analytically in its own right -- in particular, it almost belongs to the Selberg class, but not quite.

In this question I will discuss some analytic questions that arise when studying this zeta function. Where are the zeroes located? Does it satisfy the Riemann hypothesis? (Answer: no.) Does it have infinitely many zeroes on the critical line? Does it have any nice relationships with other zeta functions? And can we bound its coefficients in any useful way?