One-level density in one-parameter families of elliptic curves with non-zero average root number

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We present in this talk a (conjectural) formula for the one-level density of general one-parameter families of elliptic curves, in term of n, the rank of E over Q(t) and the average root number WE over the family. In the general case, WE is zero, and the one-level density is given by orthogonal symmetries as predicted by the conjectures of Katz and Sarnak. In the exceptional cases where WE 6= 0, we find that the statistics are given by a weighted sum of even orthogonal and odd orthogonal symmetries. The most counter-intuitive cases occur when WE = ±1. In that case, the one-level density exhibits even orthogonal symmetries when $(-1)^n WE = 1$ and odd orthogonal symmetries when $(-1)^n WE = -1$, and there is a shift of the symmetries (between orthogonal odd and orthogonal even) when n is odd.

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