Topological Combinatorics of Posets and Stratified Spaces: discrete Morse theory

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We will discuss techniques from topological combinatorics including Moebius functions, shellability and discrete Morse theory. A running theme will be how partially ordered sets (posets), and particularly their edge labelings, provide a combinatorial bridge from algebra to topology, one that translates algebraic relations into topological structure on associated simplicial complexes and chain complexes. We will touch upon most (if not all) of the following applications of poset topology: construction of small free resolutions; cohomology of subspace arrangement complements via intersection posets; combinatorics of posets of closure relations; using Moebius functions to calculate characters; characterization results in finite group theory; a complexity theory lower bound via Moebius functions; and a crystal operator analogue of Matsumoto's Theorem (namely the result that any two reduced expressions for the same Coxeter group element are connected by braid moves). Then we will turn things around and discuss some intriguing stratified spaces coming out of work of Lusztig, Fomin and others on total positivity theory and the theory of canonical bases, giving some indication of how topological-combinatorial analysis of partially ordered sets and of maps from simplices to these stratified spaces enables a determination of homeomorphism type in a manner that also sheds light on fibers and thereby on relations amongst exponentiated Chevalley generators.