Configuration theorems and skewers

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The skewer of a pair of skew lines in space is their common perpendicular. To configuration theorems of plane projective geometry involving points and lines (such as Pappus or Desargues) there correspond configuration theorems in space: points and lines in the plane are replaced by lines is space, the incidence between a line and a point translates as the intersection of two lines at right angle, and the operations of connecting two points by a line or by intersecting two lines at a point translate as taking the skewer of two lines. These configuration theorems hold in elliptic, Euclidean, and hyperbolic geometries.

This correspondence principle extends to plane configuration theorems involving polarity. For example, the theorem that the three altitudes of a triangle are concurrent corresponds to the Petersen-Morley theorem that the common normals of the opposite sides of a space right-angled hexagon have a common normal.

I shall discuss two approaches to the proof, an elliptic and a hyperbolic one. I shall also discuss the skewer versions of the Sylvester problem: given a finite collection of pairwise skew lines such that the skewer of any pair intersects at least one other line at right angle, do all the line have to share a skewer? The answer is positive in the elliptic and Euclidean geometries, but negative in the hyperbolic one.