

Higher-dimensional Expansion and Topological Overlap

Uli Wagner, IST, Austria

We survey ideas and recent developments in the theory of higher-dimensional coboundary expansion of simplicial complexes, a notion that arose in the work of Linial and Meshulam and of Gromov and that generalizes edge expansion of graphs and is defined in terms of cellular cochains of the complex.

In particular, we will give a simple proof (obtained in joint work with Dominic Dotterrer and Tali Kaufman) of Gromov's Topological Overlap Theorem: If a finite simplicial complex X is a coboundary expander in dimensions 1 up to d , then X has the following topological overlap property: for every continuous map from X to d -dimensional Euclidean space, there exists an image point p contained in the images of a positive fraction $\mu > 0$ of the d -simplices of X . More generally, the conclusion holds if d is replaced by any d -dimensional piecewise-linear manifold M , with a constant μ that depends only on d and on the expansion properties of X , but not on M .