

Points, lines, planes, etc.

June Huh, IAS and Princeton University

One of the earliest results in enumerative combinatorial geometry is the following theorem of de Bruijn and Erdős: Every set of points E in a projective plane determines at least $|E|$ lines, unless all the points are contained in a line. Motzkin and others extended the result to higher dimensions, who showed that every set of points E in a projective space determines at least $|E|$ hyperplanes, unless all the points are contained in a hyperplane. Let E be a spanning subset of a d -dimensional vector space. We show that, in the poset of subspaces spanned by subsets of E , there are at least as many $(d-k)$ -dimensional subspaces as there are k -dimensional subspaces, for every k at most $d/2$. This confirms the “top-heavy” conjecture of Dowling and Wilson from 1974 for all matroids realizable over some field. The proof relies on the decomposition theorem package for l -adic intersection complexes.

This is a joint work with Botong Wang.