Inverse problems for hyperbolic equations and artificial point sources

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We consider uniqueness results for inverse problems for hyperbolic equations in an anisotropic medium. Our aim is to determine the Riemannian metric, associated to travel times of waves, inside a domain from the observations done on the boundary.

The inverse problems in anisotropic media are not generally uniquely solvable: A change of coordinates changes the equation but does not change the boundary data. To prove uniqueness results, one may consider properties that are invariant in diffeomorphisms and aim to reconstruct those uniquely. For example, there is an underlying manifold structure that can be uniquely determined. Thus the inverse problem in a subset of the Euclidean space can solved in two steps. The first one is to reformulate the problem in terms of manifolds and to reconstruct the underlying manifold structure. The second step is to find an embedding of the constructed manifold to the Euclidean space. In the talk we focus to the reconstruction of the invariant manifold structure.

We consider solutions of hyperbolic inverse problems that are based on focusing of waves. For linear equations we consider the modified time reversal iteration where one focuses waves in an unknown medium [1,2,3]. For non-linear equations we consider the artificial point source method that applies the non-linear interaction of spherical waves or distorted plane waves to create points sources inside the medium [5,6]. The new feature of the artificial point source method is that it utilises the non-linearity as a tool in imaging.

The above methods reduce the inverse boundary value problems to passive imaging problems where one observes waves coming from the point sources that are inside the medium, and these problems are solved using geometric methods [4,5,7].

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