On the seismic inverse problem: uniqueness, stability and reconstruction

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Reflection seismology is an imaging technique used to determine properties of the earth's subsurface from reflected seismic waves generated by controlled sources like, for example, vibroesis. Models of seismic wave propagation lead to pdes or systems of pdes where the goal is to identify the coefficients containing information on the mechanical properties of rock (density, stiffness) from overdetermined data. Thus, reflection seismology imaging translates mathematically in terms of an inverse boundary value problem for pdes. In my talk, I will focus the attention on the case of time harmonic elastic waves. Seismic data from land aquisition can be represented by the Neumann to Dirichlet map. So, we end up with an inverse problem for the time harmonic elastic wave equation where, assuming small-amplitude deformations and isotropy, one seeks to identify the Lam´e parameters and the density from knowledge of the Neumann to Dirichlet or equivalently of the Dirichlet to Neumann map. First, I will overview the main known results concerning uniqueness. In the second part of the talk I will concentrate on the issue of continuous dependence, crucial for effective reconstruction. I will illustrate the results contained in [1] where Lipschitz continuous de-pendence estimates have been derived in the case of piecewise constant elasticity isotropic tensor and density on a known polyhedral partition of the background medium and I will give some highlights on the main tools of the proof. In the last part of the talk, I will describe an iterative reconstruction algorithm based on the minimization of a suitable misfit functional and I will show some numerical results that assess the effectiveness of the algorithm in identifying the unkown parameters, [2]. Finally, I will briefly overview some preliminary results (obtained in the time harmonic acoustic case,[3]) concerning the stable Lipschitz recovery of polyhedral interfaces from boundary data and I will point out the main difficulties in extending such a result to the elastic framework. References [1] E. Beretta, M. V. de Hoop, E. Francini, S. Vessella, J Zhai" Conditional Lipschitz stability of an inverse boundary value problem for the time harmonic elastic waves." Vol 33 Number 3, (2017) Inverse Problems 2] J. Shi, E. Beretta, M. V. de Hoop, E. Francini, S. Vessella," Multi-parameter iterative reconstruction with the multi-frequency Neumann-to-Dirichlet map as the data" submitted to Geophys. J. Int. (2017) 3] E. Beretta, M. V. de Hoop, E. Francini, S. Vessella "" Lipschitz determination of interfaces in the Helmholtz equation from boundary data." Comm. PDE 40, (2015) no 7 13651392