

## **On the stability and accuracy of fast structured direct solvers**

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Fast structured direct solvers have been shown to be very effective in solving large linear systems such as those from seismic inversion. On the other hand, the backward stability has rarely been studied. In this work, we investigate the numerical stability and error propagation of some fast structured direct solvers. The study is done in terms of hierarchically semiseparable (HSS) algorithms, and can be extended to more general cases. The algorithms include factorization, solution, multiplication, etc. Concrete backward error bounds are given, including a structured backward error for the solution in terms of the structured factors.

We show that, in general, the algorithms are not only faster, but also have much better stability as compared with the corresponding standard matrix algorithms. The error propagation factors only involve low-degree powers of the relevant off-diagonal numerical rank bound and the logarithm of the matrix size.

We also show that factorization-based structured solutions are usually preferred, while inversion-based ones may suffer from numerical instability. The analysis builds a comprehensive framework for understanding the backward stability of hierarchical rank structured methods. The error propagation patterns also provide insights into the improvement of other types of structured solvers and the design of new stable hierarchical structured algorithms. This is joint work with Yuanzhe Xi.