**On Convexity of Polynomials over a Box**

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In the first and main part of this talk, I show that unless P=NP, there exists no polynomial time (or even pseudo-polynomial time) algorithm that can test whether a cubic polynomial is convex over a box. This result is minimal in the degree of the polynomial and in some sense justifies why convexity detection in nonlinear optimization solvers is limited to quadratic functions or functions with special structure. As a byproduct, the proof shows that the problem of testing whether all matrices in an interval family are positive semidefinite is strongly NP-hard. This problem, which was previously shown to be (weakly) NP-hard by Nemirovski, is of independent interest in the theory of robust control. I will explain the differences between weak and strong NP-hardness clearly and show how our proof bypasses a step in Nemirovski's reduction that involves "matrix inversion". Indeed, while this operation takes polynomial time, it can result in an exponential increase in the numerical value of the rational numbers involved.

In the second and shorter part of the talk, I present sum-of-squares-based semidefinite relaxations for detecting or imposing convexity of polynomials over a box. I do this in the context of the convex regression problem in statistics. I also show the power of this semidefinite relaxation in approximating any twice continuously differentiable function that is convex over a box.

Joint work with A.A. Ahmadi (first part) and with A.A. Ahmadi and M. Curmei (second part).