Pythagoras numbers of projective varieties

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Let $X\subseteq\mathbb{P}^n$ be a non-degenerate projective variety defined over the reals. The Pythagoras number of X is the smallest integer k such that every sum-of-squares of linear forms in $\mathbb{P}^n$ can be written, on X, using at most k squares.

Determining this quantity is a problem of considerable intrinsic interest. Moreover, upper bounds on this quantity are useful for non-linear (i.e. Burer-Monteiro type) methods for sums-of-squares programs on varieties.

In this talk I will present ongoing joint work with G. Blekherman, R. Sinn and G.G.Smith where we find computable upper and lower bounds for Pythagoras numbers. These bounds arise from unexpected connections between pythagoras numbers, free resolutions and varieties of minimal degree. Moreover these bounds allow us to classify real projective varieties with small Pythagoras number relative to their dimension.