

Towards algorithms for analyzing the topology of semi-algebraic sets

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We review recent algorithmic and software developments towards the problems of deciding connectivity queries in semi-algebraic sets and computing their volumes. These are motivated by several applications in e.g. medical imaging or robotics.

We first use the critical point method, combined with symbolic analysis and computations of periods, we describe an algorithm which computes the volumes of semi-algebraic sets up to arbitrary absolute precision.

Next, based on an interplay of the critical point method with semi-algebraic versions of Ehresmann's fibration theorem, we first show how to design new algorithmic patterns for answering connectivity queries. Next, using ad-hoc algebraic elimination techniques, we derive an algorithm dedicated to smooth real algebraic sets defined by quadrics. This one runs in time subexponential in the dimension of the ambient space (but exponential in the co-dimension).

This talk is based on joint works with É. Schost (U. Waterloo) and P. Lairez (Inria).