Invariants of even ternary forms under the orthogonal group as biomarkers

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I shall report on progress made for extending the capabilities of diffusion magnetic resonnance imaging to provide biomarkers for diseases of the brain. Relevant biomarkers have been identified as invariants of a ternary quadratic form in the prevalent aquisition model, diffusion tensor imaging. As higher order aquisition models emerge, the question of associated biomarkers arise. The best candidates for clinicians to chose from are invariants of ternary quartics, sextics, ... under the orthogonal group. In order to select among these invariants, one needs to also offer a solution to an inverse problem : exhibit a ternary form corresponding to given values of the invariants.

We determine a generating set of rational invariants of the orthogonal group acting on even degree forms. Uncommonly in computational invariant theory, we do not provide complete explicit forms for the invariants, whose practical use could be unreliable, but rather provide a robust numerical algorithm to evaluate them. Each rational invariant is determined by its restriction to a *slice*. These restrictions are three-nomials invariant under the octahedral group. The exhibited set of generators furthermore allows us to solve the inverse problem and the rewriting.

Central in obtaining the invariants for higher degree forms is the preliminary construction, with explicit formulae, for a basis of harmonic polynomials with octahedral symmetry, different, though related, to cubic harmonics.

This is joint work with Paul Görlach (now at MPI Leipzig), in a joint project with Téo Papadopoulo (Inria Méditerranée)

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