Maximum Likelihood Estimation of Totally Positive Densities

Elina Robeva, Massachusetts Institute of Technology

Nonparametric density estimation is a challenging problem in theoretical statistics -- in general the maximum likelihood estimate (MLE) does not even exist! Introducing shape constraints allows a path forward. In this talk I will discuss non-parametric density estimation under total positivity (i.e. log-supermodularity) and log-concavity. I will first show that though they possess very special structure, totally positive random variables are quite common in real world data and possess appealing mathematical properties. Given i.i.d. samples from a totally positive distribution, we prove that the maximum likelihood estimator exists with probability one assuming there are at least 3 samples. We characterize the domain of the MLE and show that it is in general larger than the convex hull of the observations. If the observations are 2-dimensional or binary, we show that the logarithm of the MLE is a tent function (i.e. a piecewise linear function) with "poles" at the observations, and we show that a certain convex program can find it. In the general case the MLE is more complicated. We give necessary and sufficient conditions for a tent function to be concave and supermodular, which characterizes all the possible candidates for the MLE in the general case.