Newton Polytopes and Relative Entropy Optimization

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We describe how relative entropy is uniquely suited for optimization of signomials and sparse polynomials. The particular approach is by way of nonnegativity certificates based on the arithmetic-geometric inequality, and it connects to results by Descartes (the rule of signs) and Khovanskii (the theory of fewnomials). The facial structure of the underlying Newton polytopes plays a prominent role in our analysis. Our results have consequences in two directions. In one direction, we highlight the utility of the signomial perspective for sparse polynomial optimization. In the other direction, signomials represent a natural generalization of polynomials for which Newton polytopes continue to yield valuable insights. Our techniques provide an exponential runtime improvement on the state-of-the-art for high-degree polynomial optimization, and lead to a "Hilbert table" for signomial (and sparse polynomial) nonnegativity.