

Stratifying Multiparameter Persistent Homology (joint with H. Harrington, N. Otter, U. Tillmann)
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A fundamental tool in topological data analysis is persistent homology, which allows extraction of information from complex datasets in a robust way. Persistent homology assigns a module over a principal ideal domain to a one-parameter family of spaces obtained from the data. In applications data often depend on several parameters, and in this case one is interested in studying the persistent homology of a multiparameter family of spaces associated to the data. While the theory of persistent homology for one-parameter families is well-understood, the situation for multiparameter families is more delicate. Following Carlsson and Zomorodian we recast the problem in the setting of multigraded algebra, and we propose multigraded Hilbert series, multigraded associated primes and local cohomology as invariants for studying multiparameter persistent homology. Multigraded associated primes provide a stratification of the region where a multigraded module does not vanish, while multigraded Hilbert series and local cohomology give a measure of the size of components of the module supported on different strata. These invariants generalize in a suitable sense the invariant for the one-parameter case.