Numerical computation of the homology of semialgebraic sets

Peter Burgisser, Technical University Berlin

joint work with Felipe Cucker, Pierre Lairez, and Josue Tonelli-Cueto

We begin with a review of complexity results in real algebraic geometry. Then we describe a grid based numerical algorithm for computing the homology groups of basic closed semialgebraic sets. The idea is to construct a union of balls that retracts to the given semialgebraic sets. The homology of the union of balls can then be computed via the nerve lemma. The key parameters are the radius of the balls and the required mesh size of the grids. It is possible to bound these parameters in terms of a single quantity: the real condition number of the (description of the) semialgebraic set. This allows to bound the running time in terms of the condition number. We can prove that outside a subset of data having exponentially small measure, the cost of the algorithm is single exponential in the size of the data.

All previous algorithms solving this problem have doubly exponential complexity (and this is so for almost all input data).

arXiv:1706.07473 (Accepted for J. ACM)
arXiv:1807.06435