

## **The $n$ body and Cayley-Menger determinants**

Peter J. Olver, School of Mathematics- University of Minnesota

I will prove two recent conjectures concerning the  $n$  body matrix that appears in recent work of Turbiner, Miller, and Escobar-Ruiz on the classical and quantum  $n$  body problem. First, whenever the masses are in a nonsingular configuration, meaning that they do not lie on a lower dimensional affine subspace, the  $n$  body matrix is positive definite, and hence defines a Riemannian metric on the space coordinatized by the interpoint distances. Secondly, its determinant can be factored into the product of the order  $n$  Cayley-Menger determinant and a mass-dependent factor that is of one sign on all nonsingular mass configurations. Characterizing the latter factor and understanding the geometry of the underlying Riemannian space remain open problems.

Based on joint work with Darij Grinberg.