Shape from Metric

Albert Chern, TU Berlin

We study the isometric immersion problem: given a triangle mesh with only lengths of edges known, map the mesh to 3D as an immersed surface that realizes the intrinsic lengths. Classical approaches involve variational problems resembling stiff membrane elasticity. The challenge remains as these methods can yield surfaces that are pinched and tangled. To address this challenge, we develop a discrete theory for surface immersions into 3D. In particular, the theory correctly represents the topology of the space of immersions, i.e., the regular homotopy classes. Our approach relies on spinors to represent 3D orientations and to encode, in the spin connection, the regular homotopy class. With this theory incorporated, we resolve the challenge of pinched surfaces and ensures immersions. We demonstrate our algorithm with several applications from mathematical visualization and art directed isometric shape deformation, which mimics the behavior of thin materials with high membrane stiffness.