

Wallpaper and Rings of Integers in Quadratic Number Fields

Frank A. Farris, Santa Clara University

In my long history with 2D Euclidean wallpaper, I keep being surprised to find rings of integers of quadratic number fields popping up in situations where I thought analysis (PDEs) was the heart of the matter. I'll tell three stories:

1) When I noticed some "local symmetry" in a wallpaper pattern I'd made using functions I call wallpaper waves, it turned out that I'd accidentally chosen frequencies from an ideal in the Eisenstein integers. 2) When I went to make Fibonacci spirals to strobe in the style of John Edmark's Blooms, I came across a family of complex Fibonacci sequences in rings like the Gaussian and Eisenstein integers. 3) Finally, when I tried to create wallpaper with 5-fold rotational symmetry, which of course doesn't exist, the story involved a ring of integers with arbitrarily small elements, which is part of the reason this kind of wallpaper is impossible. ("You call these integers?")