

Apollonian Structure in the Abelian Sandpile

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The Abelian sandpile is a chip-firing game on the integer lattice which can be viewed as a simple deterministic analog to stochastic diffusion processes based on random walks. In contrast to its stochastic counterparts, the sandpile produces striking fractal scaling limits, and in this talk, we will discuss the framework which can be used to explain this fractal behavior. The heart of our results is a surprising connection between the integer superharmonic functions on the lattice which govern this PDE and Apollonian circle packings of the plane. We will try to demonstrate the crucial role that visualization played in the discovery of this connection, and consider the analogous problem which arises when attempting to develop corresponding results on other lattices.