Optimal Geometry as Art

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Topology studies those properties of curves or surfaces that are unchanged under deformation, while geometry studies properties of particular shapes. For any topological object, we can ask for an optimal geometric shape, minimizing some geometric energy. A classical example is a soap bubble which is round because it minimizes surface area while enclosing a fixed volume.

Other examples, at the frontier of current mathematical research, include knots tied tight in thick rope, which minimize their length, and surfaces which minimize elastic bending energy. The resulting shapes are not only mathematically elegant, but often exhibit striking visual beauty.

I will show two computer-generated videos, illustrating optimal shapes for knots and a mathematical way to turn a sphere inside out (controlled by surface bending energy). I will discuss the artistic choices that went into the making these films, and will show other examples of mathematical art arising from optimal geometry, including computer-generated sculpture.