Singularity Formation in General Relativity

Speck, Jared- Vanderbilt University

The celebrated theorems of Hawking and Penrose show that under appropriate assumptions on the matter model, a large, open set of initial data for Einstein's equations lead to geodesically incomplete solutions. However, these theorems are “soft” in that they do not yield any information about the nature of the incompleteness, leaving open the possibilities that i) it is tied to the blowup of some invariant quantity (such as curvature) or ii) it is due to a more sinister phenomenon, such as incompleteness due to lack of information for how to uniquely continue the solution (this is roughly known as the formation of a Cauchy horizon). In various works, some joint with I. Rodnianski and/or G. Fournodavlos, we have obtained the first rigorous results without symmetry assumptions that reveal the nature of the breakdown for solutions arising from open sets of regular initial data. More precisely, our results apply to perturbations of generalized Kasner solutions whose exponents satisfy certain assumptions, and we showed that the incompleteness is caused by Big Bang-type curvature blowup, i.e., the Kasner Big Bang is dynamically stable. In this talk, I will provide an overview of these results and explain how they are tied to some of the main themes of investigation by the mathematical general relativity community, including the remarkable recent work of Dafermos-Luk on the stability of Kerr Cauchy horizons. I will also discuss the role of geometric and gauge considerations in the proofs, as well as intriguing connections to other problems concerning stable singularity formation.