Towards energy-stable and conservative discontinuous Galerkin spectral element methods for Einstein's equations of general relativity in second order form

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Numerical methods for solving the Einstein's equations of general relativity are often derived for the first order systems, by first rewriting the underlying system of second order partial differential equations (PDE) as a system of first order hyperbolic PDEs.The reduction to first order system has the potential to introduce some numerical and computational inefficiencies, as well as numerical stability issues. There are several benefits for solving the equations in second order form.

However, it appears to be more difficult to guarantee numerical stability for the naturally second order systems than for first order reductions of them.

In this presentation we take a first, but an important, step towards designing of provably stable and high-accurate discontinuous Galerkin spectral element methods for the Einstein's equations of general relativity in second order form.

I will present the initial developments of an efficient and robust numerical method for the Einstein's equations modelling gravitational waves in second order form.

The new method combines the advantages and central idea of two very successful numerical techniques, the summation-by-parts finite difference methods and the discontinuous Galerkin methods.

We consider a 1D model problem, the shifted wave in second order form in one space dimension, and derive compatible derivative operators and numerical fluxes for the different components of the PDE flux terms.

The numerical flux allows for the derivation of a provably energy-conserving and arbitrarily high-order accurate discontinuous Galerkin spectral element method for the 1D shifted wave in second order form.

The method can be extended to higher space dimensions using tensor product elements.

I will present the proof of numerical stability and numerical experiments verifying the accuracy and stability properties of the method.