Arithmetic Properties of Dominant Self-Maps of $\mathbb{P}^N$

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Abstract: Let $f : \mathbb{P}^N \dashrightarrow \mathbb{P}^N$ be a dominant rational map of degree d defined over $\bar{\mathbb{Q}}$. Then the Weil height $h : \mathbb{P}^N(\bar{\mathbb{Q}}) \to [0,\infty)$ satisfies an upper bound

$$h\bigl(f(P)\bigr) \le \deg(f)h(P) + O(1),$$

but the corresponding lower bound is not true in general.

Part I: The *height ratio of* f,

$$\mu(f) = \limsup_{\emptyset \ne U \subset \mathbb{P}^N} \liminf_{\substack{P \in U(\bar{\mathbb{Q}}) \\ h(P)\to\infty}} \frac{h\bigl(f(P)\bigr)}{h(P)},$$

measures the generic failure of the lower bound for $h\bigl(f(P)\bigr)$. I will discuss the elementary proof that $\mu(f)$ is strictly positive, and the deeper result that is $\mu(f)$ bounded below by a constant that depends only on N and d, independent of the map f.

Part II: I will describe various properties and relations, both known and conjectural, between the (first) dynamical degree of f,

$$\delta(f) = \lim_{n\to\infty} \deg(f^n)^{1/n},$$

and the arithmetic degree of f at a point $P \in \mathbb{P}^N(\bar{\mathbb{Q}})$,

$$\alpha(f,P) = \limsup_{n\to\infty} h\bigl(f^n(P)\bigr)^{1/n}.$$