Combinatorial Representation Theory: Alcove-path models and Macdonald polynomials

Cristian Lenart, University at Albany, SUNY

This two-part talk is devoted to paths models in the representation theory of Lie algebras (I will refer mostly to the semisimple ones) and their applications. These models realize the crystals of the corresponding quantum group representations (which describe the action of the Chevalley generators on the canonical basis when the quantum parameter goes to 0). I focus on the alcove model, developed in joint work with Alex Postnikov, which is a discrete version of the celebrated Littelmann path model. In the _rst talk, I present the main constructions related to the alcove model, and illustrate them with examples and a Sage demonstration based on the implementation of this model due to Brant Jones. The second talk focuses on applications of the alcove model to: Chevalley-type multiplication formulas in the (torus equivariant) K-theory of ag varieties, Macdonald polynomials and their specializations (based on the Ram-Yip formula for these polynomials), as well as Kirillov-Reshetikhin crystals (including the corresponding energy function and combinatorial R-matrix). Some of these applications are based on joint work with Arthur Lubovsky, Satoshi Naito, Alex Postnikov, Daisuke Sagaki, Anne Schilling, and Mark Shimozono.