The geometry of Whittaker functions
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It appears that asymptotic behavior of matrix coefficients of a representation of a p-adic group is governed by geometric objects in the dual group. Twenty years ago I found evidence for this when the representation is both Iwahori-spherical and generic. In this setting the matrix coefficients are Whittaker functions, defined on T x X, where T is a maximal torus in the dual group (parameter space for these representations) and X is the character group of T (domain of the matrix coefficients).

For fixed t in T, the Whittaker function W(t,x) blows up according to certain polynomials in x which turn out (after a long calculation) to be related to the varieties used by Kazhdan-Lusztig to construct Iwahori-spherical representations starting on the dual side. Thus, one can recover the KL varieties (hence the Langlands parameter) from the representation, via asymptotics.

On the other hand, a simple calculation shows that W(t,x), as a function of t, is equal to the Lefschetz trace of t on del-bar cohomology of the line bundle associated to x. Atiyah-Singer, generalizing Hirzebruch-Riemann-Roch, proved a Lefschetz formula that may allow one to pass directly from the del-bar interpretation of W(t, ?) to the KL-interpretation of W(?,x) more conceptually, while avoiding my long calculations of yore.

In two lectures, I will explain all of the above, and report on the current state of affairs.