q-deformed Whittaker functions and the local Langlands correspondence.

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In 1967 Langlands (in his letter to Godement) conjectured an explicit formula, identifying the p-adic class one G-Whittaker function with a character of finite-dimensional G^-module of the dual group G^. This formula was provled by Shintani for GL(N), and by Casselman and Shalika for arbitrary reductive quasisplit G.
In my talk I introduce the class one q-deformed Whittaker functions, which are certain specializations of the Macdonald polynomials. Then I propose the q-analog of the Langlands-Shintani (LS) formula; it identifies the q-deformed GL(N)-Whittaker function with a character of a (finite-dimensional) Demazure module of affine algebra \hat{gl}(N), and specializing q to the power of uniformizing parameter p leads to the original (p-adic) LS formula. On the other hand, under the limit q->1 the q-analog of the LS formula provides an identification of the parabolic GL(N,R)-Whittaker function (which is an extention of the standard GL(N,R)-Whittaker function, associated with the complete flag variety GL(N)/B, to general GL(N)/P) with an S^1xU(N)-equivariant volume of the space of holomorphic maps from a two-dimensional disk into the (partial) flag variety GL(N,C)/P; the corresponding explicit formula for (parabolic) Whittaker function should be considered as the Archimedean counterpart of the Langlands-Shintani formula.