

**Atkin and Swinnerton-Dyer congruences for noncongruence modular forms**  
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Unlike their congruence counterpart, the arithmetic for noncongruence modular forms remains mysterious. A main reason is the lack of efficient Hecke operators. Absent of tools, the progress on noncongruence modular forms has been guided by numerical examples. The first systematic study of the subject was due to Atkin and Swinnerton-Dyer, who made an amazing observation that at good primes, the space of weight  $k$  cusp forms for a given noncongruence subgroup has a basis whose Fourier coefficients satisfy 3-term congruence relations, called ASD congruences, analogous to the 3-term recursive relation satisfied by a Hecke eigenform. In this survey talk we shall review the development and discuss the state-of-the-art of the ASD congruences.