

The Iwahori-Hecke Algebra, the Ramanujan Conjecture, and Expander Graphs

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Expander graphs are well-connected yet sparse graphs. The expansion property of a regular graph is governed by the second largest eigenvalue of the adjacency matrix. One can consider quotients of the Bruhat-Tits building of $GL(n)$, $n=2,3$, over a p -adic field and view them as graphs. In this context the relationship between regular expander graphs and the Ramanujan Conjecture is well understood and has led to the definition and construction of asymptotically optimal regular expanders called Ramanujan graphs. The notion of Ramanujan graph can be extended to bigraphs (i.e., biregular, bipartite graphs). In this talk I will use the representation theory of $SU(3)$ over a p -adic field to investigate whether certain quotients of the associated Bruhat-Tits tree are Ramanujan bigraphs. I will use the Bernstein presentation of the Iwahori-Hecke algebra to show that a quotient of the Bruhat-Tits tree associated with a quasisplit form G of $SU(3)$ is Ramanujan if and only if G satisfies a Ramanujan type conjecture. (This is joint work with Dan Ciubotaru).