Near optimal deterministic volume estimation via M-ellipsoids

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The M-ellipsoid, a classical tool in convex geometry, describes an ellipsoid which approximates an n-dimensional convex body K from the perspective of covering. More precisely, an M-ellipsoid E for K satisfies that 2^{O(n)} translates of E suffice to cover K and vice versa.

Following a construction of Milman, we give a deterministic 2^{O(n)} time and poly(n) space algorithm for constructing an M-ellipsoid for any convex body. Our algorithm consists of solving a sequence of convex programs over semidefinite matrices, which implement Milman's technique of "isomorphic symmetrization".

As applications of this algorithm (and variants), we give the first deterministic (1+1/eps)^{O(n)} time algorithm for estimating the volume of a convex body up to a (1+eps)^n factor in the oracle model (nearly matching a lower bound of Barany and Furedi), and the first deterministic single exponential time algorithm for solving the shortest vector problem in lattices under general norms.