On the existence of 0/1 polytopes with high semidefinite extension complexity

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In Rothvoss it was shown that there exists a 0/1 polytope (a polytope whose vertices are in $\{0,1\}^{n}$) such that any higher-dimensional polytope projecting to it must have $2^{\Omega(n)}$ facets, i.e., its linear extension complexity is exponential. The question whether there exists a 0/1 polytope with high PSD extension complexity was left open. We answer this question in the affirmative by showing that there is a 0/1 polytope such that any spectrahedron projecting to it must be the intersection of a semidefinite cone of dimension $\sim 2^{\Omega(n)}$ and an affine space. Our proof relies on a new technique to rescale semidefinite factorizations. We also show the existence of a polygon with d integral vertices and semidefinite extension complexity $\Omega((d/ \log d)^{1/4})$.

Joint work with J. Briet and D. Dadush.