Improved bounds and algorithms for graph cuts and network reliability

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Karger developed the first fully-polynomial approximation scheme to estimate the probability that a graph G becomes disconnected if its edges are removed independently with given probabilities. This algorithm runs in $O(n^{5+o(1)} \epsilon^{-3})$ time to obtain an estimate within relative error ϵ.

We improve this runtime in two key ways, one algorithmic and one graph-theoretic. The main (graph-theoretic) improvement hinges on better bounds on the number of edge cuts which are likely to fail. We describe a new graph parameter, which simultaneously influences all the bounds used by Karger, and use it to obtain much tighter estimates of the behavior of the cuts of G. These techniques allow us to improve the runtime to $n^{3+o(1)} \epsilon^{-2}$, which is essentially best-possible for the meta-approach proposed by Karger; our results also rigorously prove certain experimental observations of Karger and Tai. A key driver of Karger's approach is his seminal bound on the number of small cuts: we also show how to improve this when the min-cut size is ``small'' and odd. A key technical ingredient of our work is a simple new way in which differential-equation approximations can help analyze randomized algorithms.

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