Spectral partitioning and higher-order Cheeger inequalities

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A basic fact in spectral graph theory is that the number of connected components in an undirected graph is equal to the multiplicity of the eigenvalue zero in the Laplacian matrix of the graph. In particular, the graph is disconnected if and only if there are at least two eigenvalues equal to zero. Cheeger's inequality and its variants provide a robust version of the latter fact; they state that a graph has a sparse cut if and only if there are at least two eigenvalues that are close to zero.

It has been conjectured that an analogous characterization holds for higher multiplicities, i.e., there are k eigenvalues close to zero if and only if the vertex set can be partitioned into k subsets, each defining a sparse cut. We resolve this conjecture positively. These techniques are extended to exhibit a nearly optimal quantitative connection between the expansion of small sets and the spectrum of the graph. A key technical tool from the theory of metric embeddings is the use of random metric partitions to smoothly localize eigenfunctions.

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