Diffuse Scattering on Graphs and Combinatorial Inverse Problems

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Spectral graph theory is a rich and well-developed theory for both the combinatorial and analytic properties of graphs. Typically, people study the graph analog of the Poisson equation

$$Lu = f$$

where L is the combinatorial or graph Laplacian. We consider the effect of introducing inhomogeneities on the vertices, as represented by the addition of a (vertex) potential term to the above equation. We call this problem the problem of diffuse scattering on graphs because of its analogy to related problem in the continuous model. We show how to construct solutions in the presence of weak scatterers from the solution to the homogeneous (background problem) using Born series, providing necessary conditions for convergence and demonstrating the process through numerous examples. In addition, we outline a method for finding Green's functions for Cayley graphs for both abelian and non-abelian groups. We also discuss the effects of sparsity on our method and results, outlining the simplifications that can be made provided that the scatterers are weak and well-separated. Finally, we discuss preliminary work on inverse problems: given boundary data, can we infer the properties of the internal vertices.

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