Harmonic pinnacles in the Discrete Gaussian model

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The 2D Discrete Gaussian is the crystal surface model which gives each height function $\eta: Z^2 \to Z$ a probability proportional to $\exp[-\beta H(\eta)]$, where β is the inverse-temperature and $H(\eta)=\sum(\eta_x-\eta_y)^2$ sums over nearest-neighbor bonds. We consider the model at large fixed β, where it is flat unlike its continuous analog (the Gaussian Free Field).
We first establish that the maximum height in an $L\times L$ box with 0 boundary conditions concentrates on two integers $M,M+1$ with $M \sim [(2/\pi\beta) \log L \log\log L]^{1/2}$. The key is a large deviation estimate for the height at the origin in Z^2, dominated by "harmonic pinnacles", integer approximations of a harmonic variational problem. Second, in this model conditioned on $\eta \geq 0$ (a floor), the average height rises, and in fact the height of almost all sites concentrates on levels $H,H+1$ where $H \sim M/\sqrt{2}$. This in particular pins down the asymptotics, and corrects the order, in results of Bricmont, El-Mellouki and Fröhlich (1986). Finally, our methods extend to other classical surface models (e.g., restricted SOS), featuring connections to p-harmonic analysis and alternating sign matrices.
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