Phase transitions in models of quasicrystals

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Quasicrystals are solids with an unusual molecular structure, first discovered experimentally by Shechtman et al (1984), who showed that certain metal alloys have diffraction patterns with rotational symmetries that cannot occur for crystals. Before this discovery it had been believed that all solids in thermal equilibrium were crystalline. Levine and Steinhardt (1984) proposed that clusters of atoms with a particular shape could form the "building blocks" for these quasicrystals. They argued that quasicrystals could then be modeled by aperiodic tilings (first discovered by Berger, 1966): mathematical models of shapes that can tile the plane but only in a non-periodic (i.e., non-crystalline) fashion. This would be a model at zero temperature, since no defects are allowed in a (perfect) tiling. We consider models of quasicrystals at positive temperature, allowing for defects. Some of these models exhibit a first-order phase transition between ordered and disordered phases. The transition parallels the expected solid-liquid transition of crystal-forming materials in thermal equilibrium.