Flavors of Rigidity for Sticky Spheres

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Abstract: Given a framework consisting of a finite configuration of points and fixed length bars between some pairs of the points, there are several notions of what it means for the framework to be rigid. Linearization of the distance constraints leads to the notion of infinitesimal rigidity and dually static rigidity. Second-order analysis leads to second-order rigidity, and a closely related stronger notion of prestress stability, which is a natural concept in terms of the local minimization of energy functions. However, often one wants to know about possible configurations that may not be close to the given configuration. This leads to global rigidity, where there are no other non-congruent configurations satisfying the distance constraints in the given Euclidean space. There are techniques for detecting global rigidity, but they usually assume that the configuration is generic. For the unit distance graphs coming from packings of "sticky spheres", the configuration is far from being generic. There are techniques for detecting universal rigidity, where the configuration is not only rigid in the given Euclidean space, but also in any other higher dimensional Euclidean space. Sadly, unit distance graphs are never universally rigid unless the graph is the complete graph. But for sticky spheres there is a concept of the configuration being unique up to congruences, where only embeddings of the spheres are considered. We will show examples of how some basic techniques of rigidity theory can be applied to detect such uniquely embedded sticky sphere realizations in addition to prestress stable configurations.