

The limit shape of the Abelian Sandpile identity

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The Abelian Sandpile Model is a minimalistic cellular automaton description of diffusion. Nonetheless, when studied on regular graphs, such as a square portion of the square lattice, it shows a rich structure: under a variety of simple dynamical protocols, the domain is asymptotically filled with polygonal patches of distinct periodic tilings. The shape and content of the patches on the scaled domain stabilises in the limit, becoming a fractal with a Sierpinski-like structure. The 'recurrent identity configuration' is one such protocol, of a special importance, due to its role in the underlying group structure of the model. The determination of the limit shape for this configuration was an open problem up to date, and is here solved. For example, the 'big blue square' in the middle has distance from the boundary $0.58315637\dots$, the only real root of $x(x+1/2)(x+1)=1$. More generally, all limit coordinates of the vertices of the polygonal patches can be determined explicitly, and are valued in the complex cubic field of discriminant -413 .