DISCRETE ANALOGUES IN HARMONIC ANALYSIS: A QUADRATIC CARLESON THEOREM

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Consider the discrete maximal function

$$\mathcal{C}_\Lambda f(n) := \sup_{\lambda \in \Lambda} \left| \sum_{m \neq 0} f(n-m) \frac{e(\lambda m^2)}{m} \right|$$

acting on $L^2(\mathbb{Z})$ functions, where $\Lambda \subset [0,1]$ is a set of modulation parameters. Here and throughout, $e(t) := e^{2\pi i t}$ denotes the complex exponential. In this talk we exhibit a class of infinite Λ for which the pertaining maximal function is bounded on $L^2(\mathbb{Z})$.

This result can be thought of as a (weakened) discrete analogue of E. Stein's result on the boundedness of the continuous maximal function,

$$\mathcal{C}_2 f(x) := \sup_{\lambda \in \mathbb{R}} \left| \int f(x-y) \frac{e(\lambda y^2)}{y} \, dy \right|.$$

Our method is heavily influenced by J. Bourgain's celebrated paper, *Pointwise ergodic theorems for arithmetic sets.*

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