Hausdorff dimension of singular vectors

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In this talk, I will discuss the problem of computing the Hausdorff dimension of the set of singular vectors in R^d. Singular vectors are a natural counterpart to the more familiar notion of badly approximable vectors, for which much about their Hausdorff dimension and related questions are known. In dimension one, the notion reduces to that of being rational, making it difficult to develop a good intuition for their behavior in higher dimensions. An added difficulty is the lack of a higher dimensional analog of the theory of continued fractions. In recent work joint with Nicolas Chevallier, we determined the Hausdorff dimension of the set of singular vectors to be d^2/(d+1). Our approach is guided in part by the idea of renormalization in dynamics, allowing us to fabricate what might be called a "self-similar structure" on a set that would otherwise not be considered self-similar. I will also touch upon some recent work in progress joint with Chevallier and Yann Bugeaud on the Hausdorff dimension of singular vectors with exponents that improves work of Roger Baker from the 1990's.