Measurable equidecompositions and circle squaring

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The Banach-Tarski paradox claims that any two bounded sets A, B with non-empty interiors in a three (or higher) dimensional Euclidean space are equidecomposable: we can divide A into finitely pieces, move them by isometries, and obtain a partition of B.

In the plane, Tarski's circle squaring problem from the 1920s asked whether the square is equidecomposable to a disc (of the same area, as the isometry group is amenable).

In 1990 Laczkovich solved this problem using combinatorial arguments and equidistribution theorems. He showed that for any two sets A and B in the n-dimensional Euclidean space with the same non-zero Lebesgue measure and with boundary of box dimension less than n, A is equidecomposable to B (using translations only).

I will discuss these problems and, in particular, present our recent results with Lukasz Grabowski and Oleg Pikhurko that the pieces can be made measurable in all of the above equidecompositions (provided that A and B are measurable and have equal measures).