New constructions of Kakeya and Besicovitch sets

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A planar set that contains a unit segment in every direction is called a Kakeya set (more precisely, a Besicovitch set). These sets have been studied intensively in geometric measure theory and harmonic analysis since the work of Besicovitch (1919); we find a new connection to game theory and probability. A hunter and a rabbit move on an n-vertex cycle without seeing each other until they meet. At each step, the hunter moves to a neighboring vertex or stays in place, while the rabbit is free to jump to any node. Thus they are engaged in a zero sum game, where the payoff is the capture time. We show that every rabbit strategy yields a Kakeya set. Passing to the scaling limit yields a simple construction of a random Kakeya set with zero area from two Brownian motions. I will conclude with a new, optimal, deterministic construction due to Chris Bishop, that was inspired by the random construction above.

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5586. http://arxiv.org/abs/1207.6389