On the fractal geometry of horseshoes in arbitrary dimensions

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We will discuss some works in progress which give a reasonably good perspective of understanding the main properties of the fractal geometry of typical dissipative horseshoes in arbitrary dimensions.

In the first work, in collaboration with J. Palis and M. Viana, given a horseshoe Λ whose stable spaces have dimension k we define a family of fractal dimensions (the so-called *upper stable dimensions*) $\overline{d}_s^{(j)}(\Lambda), 1 \le j \le k$ which satisfy $\overline{d}_s^{(1)}(\Lambda) \ge \overline{d}_s^{(2)}(\Lambda) \ge \cdots \ge \overline{d}_s^{(k)}(\Lambda) \ge HD(\Lambda \cap W^s(x)), \forall x \in \Lambda$ (and analogously for the unstable directions) with the following properties: given $1 \le r \le k$ and $\varepsilon > 0$ there is a $\varepsilon-$small C^∞ perturbation of the original diffeomorphism for which the hyperbolic continuation of Λ has a subhorseshoe $\tilde{\Lambda}$ which has strong stable foliations of codimensions j for $1 \le j \le r$ and which satisfies $\overline{d}_s^{(r)}(\tilde{\Lambda}) > \overline{d}_s^{(r)}(\Lambda) - \varepsilon$.

In the second work in progress, in collaboration with W. Silva (which extends a previous joint work in codimension 1), we prove that if a horseshoe Λ has strong stable foliations of codimensions j for $1 \le j \le r$ and satisfies $\overline{d}_s^{(r)}(\Lambda) > r$ then it has a small C^∞ perturbation which contains a blender of codimension k: in particular C^1 images of stable Cantor sets of it (of the type $\Lambda \cap W^s(x)$) in $\mathbb{R}^k$ will typically have persistently non-empty interior. We also expect to prove that when $r < \overline{d}_s^{(r)}(\Lambda) \le r+1$ the Hausdorff dimension of these stable Cantor sets typically coincide with $\overline{d}_s^{(r+1)}(\Lambda)$, and this dimension depends continuously on Λ on these assumptions, which would imply typical continuity of Hausdorff dimensions of stable and unstable Cantor sets of horseshoes.