Spectral gaps via additive combinatorics

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A spectral gap on a noncompact Riemannian manifold is an asymptotic strip free of resonances (poles of the meromorphic continuation of the resolvent of the Laplacian). The existence of such gap implies exponential decay of linear waves, modulo a finite dimensional space; in a related case of Pollicott--Ruelle resonances, a spectral gap gives an exponential remainder in the prime geodesic theorem.

We study spectral gaps in the classical setting of convex co-compact hyperbolic surfaces, where the trapped trajectories form a fractal set of dimension $2\delta + 1$. We obtain a spectral gap when $\delta=1/2$ (as well as for some more general cases). Using a fractal uncertainty principle, we express the size of this gap via an improved bound on the additive energy of the limit set. This improved bound relies on the fractal structure of the limit set, more precisely on its Ahlfors--David regularity, and makes it possible to calculate the size of the gap for a given surface.