In a joint work (unpublished) with D.Damanik we show that under natural technical conditions, the sum of a $C^2$ dynamically defined Cantor set with a compact set in most cases (for almost all parameters) has positive Lebesgue measure, provided that the sum of the Hausdorff dimensions of these sets exceeds one. As an application, we show that for many parameters, the Square Fibonacci Hamiltonian has spectrum of positive Lebesgue measure, while at the same time the density of states measure is purely singular. The proof uses the thermodynamic formalism, and is based on our previous results with D.Damanik and B.Solomyak that establish absolute continuity of convolutions of measures arising in hyperbolic dynamics with exact-dimensional measures.