Estimating long-term behavior of periodically driven flows without trajectory integration

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Periodically driven flows are fundamental models of chaotic behavior and the study of their transport properties is an active area of research. A well-known analytic construction is the augmentation of phase space with an additional time dimension; in this augmented space, the flow becomes autonomous or time-independent. We prove several results concerning the connections between the original time-periodic representation and the time-extended representation, focusing on transport properties. In the deterministic setting, these include single-period outflows and time-asymptotic escape rates from time-parameterized families of sets. We also consider stochastic differential equations with time-periodic advection term. In this stochastic setting one has a time-periodic generator (the differential operator given by the right-hand-side of the corresponding time-periodic Fokker-Planck equation). We define in a natural way an autonomous generator corresponding to the flow on time-extended phase space. We will state relationships between these two generator representations and use these to quantify decay rates of observables and to determine time-periodic families of sets with slow escape rate. Finally, we use the generator on the time-extended phase space to create efficient numerical schemes to implement the various theoretical constructions. In particular, no expensive time integration is required. We introduce an efficient new hybrid approach, which treats the space and time dimensions separately.

This is joint work with Peter Koltai.